

Beurling, Arne**[Carleson, Lennart; Malliavin, Paul; Neuberger, John; Wermer, John]****The collected works of Arne Beurling. Volume 1: Complex analysis. Volume 2: Harmonic analysis. Ed. by Lennart Carleson, Paul Malliavin, John Neuberger, John Wermer. (English)**

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Contemporary Mathematicians. Boston etc.: Birkhäuser Verlag. xx, 475 p./v.1; xx, 389 p./v.2 sFr. 168.00/set; DM 198.00/set (1989).

Arne Karl-August Beurling (1905-1986) was Professor at Uppsala from 1937 till 1954; since 1954, he became a Permanent Member and Professor at the Institute for Advanced Study in Princeton; he was a member of several Academies, and was awarded several Scientific Prizes. From “Arne Beurling in memoriam” by *L. Ahlfors* and *L. Carleson*, *Acta Math.* 161, 1-9 (1988), reprinted in the volumes under review, we quote “[he] was a highly creative mathematician whose legacy will influence future mathematics for many years to come, maybe even for generations.... He published very selectively..., and a sizeable part of his work has never appeared in print.”

“The work of Arne Beurling falls into three main categories: complex analysis, harmonic analysis, and potential theory. In a characteristic way he transformed all of these areas of mathematics and made them interact with each other. This unity and confluence of original ideas and methods make him unique analysts of our time”.

Arne Beurling published 46 papers in French and English (including joint papers with *L. V. Ahlfors*, *H. Helson*, *A. E. Livingstone*, *J. Deny* and *P. Malliavin*; about a quarter of his papers are published in *Acta Mathematica*). Among his doctoral students are *C.-G. Esseen*, *L. Carleson*, *G. Borg*, *B. Nyman*, and *Sonja Lyttkens*.

The two volumes (“In accordance with Beurling’s wishes, the editors have divided the papers into two parts: complex analysis and harmonic analysis.”) “The collected works of Arne Beurling” contain all of his papers (including the thesis “*Études sur un problème de majoration*”, Uppsala 1933), and, in addition, the Mittag-Leffler Lectures on Complex and Harmonic Analysis (1977-1978), written up by *L. Carleson* and *J. Wermer* (hitherto unpublished), Selected Seminars on Complex Analysis, University of Uppsala, 1938-1952, and Selected Seminars on Harmonic Analysis, University of Uppsala, 1938-1952. In the Mittag-Leffler lectures “[Beurling] described the development of his ideas in various fields of analysis”. Beurling himself was not able “to review the unpublished papers as they appear here.”

Moreover the two volumes contain the above-mentioned memorial article by *L. Ahlfors* and *L. Carleson*, and a Séminaire Bourbaki lecture “*Quotients des fonctions définies-négatives*” by *J.-P. Kahane*, describing unpublished joint work of *A. Beurling* and *J. Deny*.

In a short review it does not seem to be possible to give an adequate description of the papers collected in these two volumes. To get an impression, what is dealt with, we again quote from the paper “In memoriam Arne Beurling”.

(The thesis is) “... a whole program for research in function theory in the broadest sense. As such it has been one of the most influential mathematical publications... Beurling’s leading idea was to find new estimates for the harmonic measure by introducing concepts... which are inherently invariant under conformal mapping.” An important concept was the notion of “extremal distance”, “a forerunner of the notion of “extremal length”, which is at the basis of quasiconformal mappings and... Teichmüller theory”. His paper “*Ensembles exceptionnels*” (1940) “became the origin of numerous studies of exceptional sets and boundary behaviour of holomorphic features... Beurling’s treatment of quasi-analyticity was combined with harmonic analysis and potential theory.” Beurling’s most famous theorem as well as the definition of “inner” and “outer” functions may be found in his paper in *Acta Math.* 81 (1949); the theorem is given, for example, in *W. Rudin* “*Real and Complex Analysis*” in 17.21 as “Beurling’s theorem”.

“Beurling’s first paper in harmonic analysis is” his extension of Wiener’s proof of the prime number theorem to “generalized integers”. This paper [mentioned also in *Rudin*’s “*Functional Analysis*”] is the first one in a long series of papers on “generalized integers”, see for example *J. Knopfmacher*’s “*Abstract Analytic Number Theory*”. Furthermore he proved and emphasized the spectral radius formula, and in

a highly original manner, he dealt with the problem of approximating bounded functions ϕ by linear combinations of exponentials from the spectrum of ϕ . His papers [3] and [6] are referred to, explicitly for example in L. Loomis' "Introduction to Abstract Harmonic Analysis".

Beurling's investigations concerning duality between capacity measures and the Dirichlet integral, and concerning the importance of contractions for spectral synthesis "led him to... a new foundation of potential theory". The notion of Dirichlet space, which is summarized in the Encyclopedic Dictionary of Mathematics in 338Q, was introduced by Beurling.

Concluding, every mathematician working in harmonic analysis, theory of complex functions or potential theory, ought to be grateful to the publishers for the fact that now he has easy access to Beurling's papers.

Reviewer: [Wolfgang Schwarz \(Frankfurt/Main\)](#)

MSC:

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