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Iwasawa modules up to isomorphism. (English) Zbl 0732.11061

Algebraic number theory - in honor of K. Iwasawa, Proc. Workshop Iwasawa Theory Spec. Values L-Funct., Berkeley/CA (USA) 1987, Adv. Stud. Pure Math. 17, 171-207 (1989).

[For the entire collection see Zbl 0721.00006.]

In this interesting paper, the author proposes some methods for the study of finitely generated modules over the completed group ring $\Lambda = \mathbb{Z}_p[[G]]$ of a compact p-adic Lie group G (not necessarily commutative). A basic tool is the "homotopy theory" for such Λ- modules X, which amounts to considering them up to projective factors. Analogues of homotopy groups are certain Λ-modules $E^i(X) = Ext^i_\Lambda(X, \Lambda)$. In the case $G \simeq \mathbb{Z}_p$, the Λ-module X is determined up to isomorphism by $E^0(x) \simeq \Lambda^r$ (where $r = rank_\Lambda X$), $E^1(X)$, $E^2(X)$, and a certain class in $Ext^2_\Lambda(E^2(X), E^1(X))$. Essentially two arithmetical applications are given:

1) If k is a finite extension of $\mathbb{Q}_p$, K/k a $\mathbb{Z}_p$- extension, $G = Gal(K/k)$, M the maximal abelian pro-p-extension of K, then $X = Gal(M/K)$ is determined by $\mu_K(p)$ (= *the* group of p-power roots of 1 in K) and a canonical class $\chi \in H^2(G, \mu_K(p)^\vee)$ (where $^\vee$ denotes the Pontryagin dual).

2) If k is a finite extension of $\mathbb{Q}$, S a finite set of primes in k containing the primes above p.∞, K/k an S-ramified $\mathbb{Z}_p$- extension, K^S (resp. M^S) the maximal (resp. maximal abelian) S-ramified pro-p-extension of K, then $X_S = Gal(M^S/K)$ is determined by $W_S = E_2(p)^{Gal(K^S/K)}$ (where $E_2(p)$ is the dualizing module of $Gal(K^S/k)$, in the sense of Galois cohomology) and a canonical class $\chi \in H^2(G, W_S^\vee)$. Note that $W_S^\vee$ is related in a precise way to the Λ- torsion of X_S.

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MSC:

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11R34 Galois cohomology

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