Israfilov, D. M.
Approximation of the Riemann function by extremal polynomials. (Russian) Zbl 0732.30032
Special questions of function theory, 4, Baku, 101-122 (1989).

[For the entire collection see Zbl 0702.00015.]

Given a fixed point \( z_0 \) of a bounded simply connected domain \( G \), let \( \phi \) be the conformal mapping of \( G \) onto the disk \( |w| < r_0 \) with \( \phi(z_0) = 0, \phi'(z_0) = 1 \). For each \( p > 0 \) put \( \pi_{n,p}(z) := \int_{z_0}^{z} P_{n-1}(\zeta)d\zeta \), where \( P_{n-1} \) is a polynomial of degree \( \leq n - 1 \), minimizing the expression \( \int_G |[\phi'(\zeta)]^{2/p} - Q_{n-1}(\zeta)|^p d\sigma(\zeta) \) for all polynomials \( Q_{n-1} \) of degree \( \leq n - 1 \) with \( Q_{n-1}(z_0) = 1 \).

Main result: If \( G \) is a quasidisk then

\[
\sup_{z \in G} |\int_{z_0}^{z} [\phi'(\zeta)]^{2/p} d\zeta - \pi_{n,p}(z)| \leq c n^{-\gamma}, \quad n \geq 1, \quad p \geq 2,
\]

where \( \gamma = \gamma(G,p) \), \( c = c(G,p) \) are constants independent of \( n \). If \( p = 2 \) the result is due to V. I. Belyĭ [Mat. Sbornik, n. Ser. 102(144), 331-361 (1977; Zbl 0358.30005)].

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