Israfilov, D. M.
Approximation of the Riemann function by extremal polynomials. (Russian) [Zbl 0732.30032]
Special questions of function theory, 4, Baku, 101-122 (1989).

[For the entire collection see Zbl 0702.00015.]

Given a fixed point $z_0$ of a bounded simply connected domain $G$, let $\phi$ be the conformal mapping of $G$ onto the disk $|w| < r_0$ with $\phi(z_0) = 0$, $\phi'(z_0) = 1$. For each $p > 0$ put $\pi_{n, p}(z) := \int_{z_0}^{z} P_{n-1}(\zeta) d\zeta$, where $P_{n-1}$ is a polynomial of degree $\leq n - 1$, minimizing the expression $\int_{G} |(\phi'(z))^{2/p} - Q_{n-1}(z)|^{p} d\sigma(z)$ for all polynomials $Q_{n-1}$ of degree $\leq n - 1$ with $Q_{n-1}(z_0) = 1$.

Main result: If $G$ is a quasidisk then

$$\sup_{z \in G} \left| \int_{z_0}^{z} [\phi'(\zeta)]^{2/p} d\zeta - \pi_{n, p}(z) \right| \leq c n^{-\gamma}, \quad n \geq 1, \quad p \geq 2,$$

where $\gamma = \gamma(G, p)$, $c = c(G, p)$ are constants independent of $n$. If $p = 2$ the result is due to V. I. Belyi [Mat. Sbornik, n. Ser. 102(144), 331-361 (1977; Zbl 0358.30005)].

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MSC:

30E10 Approximation in the complex plane
41A10 Approximation by polynomials

Keywords:

Bieberbach polynomials