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Certain classes of harmonic functions with singularities on a quasiconformal arc. (English. Russian original) Zbl 0732.31003

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The present paper is devoted to the study of a problem concerned with the constructive description of classes of harmonic functions and is closely connected with studies initiated in the middle sixties by Dzyadyk. Let $L \subset \mathbb{C}$ be a bounded quasiconformal arc, and let $C_\Delta(L)$ be the class of functions which are real, continuous on $\bar{\mathbb{C}}$, and harmonic in $\bar{\mathbb{C}} \setminus L$, $U(z,\delta) = \{\zeta : |\zeta - z| \leq \delta\}$, $z \in \mathbb{C}$, $\delta > 0$.

We denote by $C^\omega_{k,\Delta}(L)$, where $\omega\ (\delta)$, $\delta > 0$ is a nondecreasing function, $\omega(+0) = 0$, the class of functions $u \in C_\Delta(L)$ satisfying the condition $\omega_{k,\Delta}(u, \delta, \mathbb{C}) \leq c\omega(\delta)$, $\forall\delta > 0$ for the k-th harmonic modulus of smoothness. We assume also that $\omega\ (\delta)$ satisfies the relation $\omega(t\delta) \leq c_1 t^k \omega(\delta)$, $\forall t > 1$, $\forall\delta > 0$.

In an earlier paper [ibid. 40, No.1, 3-7 (1988; Zbl 0699.31002)] a constructive characteristic was obtained for the classes $C^\omega_{1,\Delta}(L)$. In the present paper we generalize this result to the case of arbitrary $k > 1$.

MSC:

31A15 Potentials and capacity, harmonic measure, extremal length and related notions in two dimensions
31C05 Harmonic, subharmonic, superharmonic functions on other spaces
30E10 Approximation in the complex plane
30C62 Quasiconformal mappings in the complex plane

Keywords:

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