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Superadditivity of Fisher's information and logarithmic Sobolev inequalities. (English)

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Summary: We prove a theorem characterizing Gaussian functions and we prove a strict superadditivity property of the Fisher information. We use these results to determine the cases of equality in the logarithmic Sobolev inequality on R^n equipped with Lebesgue measure and with Gauss measure. We also prove a strengthened form of Gross's logarithmic Sobolev inequality with a "remainder term" added to the left side. Finally we show that the strict form of Gross's inequality is a direct consequence of an inequality due to Blachman and Stam, and that this in turn is a direct consequence of strict superadditivity of the Fisher information.

MSC:

60E15 Inequalities; stochastic orderings
94A15 Information theory (general)

Keywords:

superadditivity property of the Fisher information; logarithmic Sobolev inequality

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