Gąsiorek, Marcin
On algorithmic Coxeter spectral analysis of positive posets. (English) [Zbl 07323513]

Summary: Following a general framework of Coxeter spectral analysis of signed graphs \( \Delta \) and finite posets \( I \) introduced by D. Simson [SIAM J. Discrete Math. 27, No. 2, 827–854 (2013; Zbl 1272.05072)] we present efficient numerical algorithms for the Coxeter spectral study of finite posets \( I = \{\{1, \ldots, n\}, \preceq_I\} \) that are positive in the sense that the symmetric Gram matrix \( G_I := \frac{1}{2}(C_I + C_I^t) \in \mathbb{M}_n(\mathbb{Q}) \) is positive definite, where \( C_I \in \mathbb{M}_n(\mathbb{Z}) \) is the incidence matrix of \( I \) encoding the relation \( \preceq_I \). In the framework of scientific computing we present a complete Coxeter spectral classification of finite positive posets \( I \) of size \( n = |I| < 20 \). It extends one of the main results obtained in [the author et al., Eur. J. Comb. 48, 127–142 (2015; Zbl 1318.06004)] for posets of size \( n \leq 10 \). We also show that the connectivity of such posets \( I \) is determined by the complex Coxeter spectrum \( \text{specc}_I \subseteq \mathbb{C} \); equivalently, by the Coxeter polynomial \( \text{cox}_I(t) \in \mathbb{Z}[t] \od I \).

One of the main results of the paper is a new technique to compute in a polynomial time a \( \mathbb{Z} \)-invertible matrix \( B_I \in \mathbb{M}_n(\mathbb{Z}) \) such that \( B_I^t \cdot C_I \cdot B_I = \bar{G}_{D_I} \), where \( \bar{G}_{D_I} \in \mathbb{M}_n(\mathbb{Z}) \) is a non-symmetric Gram matrix of a simply-laced Dynkin diagram \( D_I \in \{D_n, E_6, E_7, E_8\} \) associated with a finite positive poset \( I \).

MSC:
00-XX General and overarching topics; collections

Keywords:
positive poset; edge-bipartite graph; spectral graph theory; Dynkin-type; Coxeter spectrum; numerical algorithm

Software:
Python; Maple

Full Text: DOI

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