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Systems of combinatory logic related to Quine’s ‘New Foundations’. (English) [Zbl 0733.03009]

The author introduces a weak system of pure combinatory logic to which are added pairing operators and an equality function. This is made part of a first-order logic with (the combinatory) equality and is called TRC. A variant TRCU of this allows for the presence of “ur-elements”. The author is able to define a weak form of bracket abstraction in TRC which requires that all occurrences of a variable to be abstracted can be assigned a certain common “type”, and for a restricted class of terms, any common “type”.

While TRC is of some interest in itself, it comes as a surprise to find that Quine’s set theory “New Foundations” can be shown to be equivalent in strength to TRC. In a similar way TRCU is shown to be equivalent to the fragment $NF + Infinity$ of NF as described by Jensen. The consistency of $NF + Infinity$ related to ZFC is known, the question of the consistency of NF remains open, but this work may provide a new means of answering the question.

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MSC:
03B40 Combinatory logic and lambda calculus
03E70 Nonclassical and second-order set theories

Keywords:
type-respecting combinators; weak system of pure combinatory logic; pairing operators; TRC; ur-elements; bracket abstraction; New Foundations

Full Text: DOI

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