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Breakdown points of affine equivariant estimators of multivariate location and covariance matrices.  (English) Zbl 0733.62058

The finite-sample replacement breakdown point of a location estimator $t_n \hat{X}$ at a collection $X$ is defined as the smallest fraction of outliers that can take the estimate over all bounds:

$$e^*(t_n, X) = \min_{1 \leq m \leq n} \left\{ \frac{m}{n} : \sup_{1 \leq m \leq n} \| t_n \hat{X} - t_n \hat{Y}_m \| = \infty, \right\}$$

where $Y_m$ is $X$ with $m$ replacements. In section 2 it is shown that $\lceil (n + 1)/2 \rceil / n$ (where $\lceil u \rceil$ denotes the nearest integer less than or equal to $u$) is an upper bound for the breakdown point of a translation equivariant location estimator, and that the same bound holds for the $L_1$-estimator. In section 3, the role of the indicated bound is considered for affine equivariant estimators of location and covariance, in particular for minimum volume ellipsoid and S-estimators. In section 4, the breakdown point is related to a measure of performance based on large deviation probabilities, and in section 5 it is shown that one-step reweighting preserves the breakdown point.

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