Johnson, Will
Dp-finite fields. I(A): The infinitesimals. (English) [Zbl 07333019]

In the article under review, the author proves that every definable valuation ring of an infinite NIP field of positive characteristic is henselian. He furthermore provides a construction of a type-definable group of infinitesimals for arbitrary dp-finite (unstable) fields, not necessarily dp-minimal (that is, of dp-rank 1).

Recall that Shelah’s conjecture states that an infinite NIP field is either separably closed, real closed or it can be equipped with a non-trivial henselian valuation. By a result of F. Jahnke and J. Koenigsmann [J. Symb. Log. 80, No. 1, 85–99 (2015; Zbl 1372.03078)], it is equivalent to stating that an infinite NIP field is either separably closed, real closed or it can be equipped with a non-trivial definable henselian valuation. Thus, a positive answer to Shelah’s conjecture yields a positive answer to the long-standing conjecture that infinite stable fields are separably closed. It is worth noting that in a recent work of the author together with Tran, Walsberg and Ye, they answered the latter conjecture positively for stable large fields.

The author has shown the existence of a non-trivial henselian valuation, whenever the field is dp-minimal, that is, of dp-rank 1, or equivalently, if it contains no indiscernible random (or ICT) pattern of depth 2. A key step in his proof was constructing a type-definable subgroup of infinitesimal elements satisfying that when translating every infinite definable set, the intersection remains infinite. The group of infinitesimals so constructed is shown to be an ideal with respect to a non-trivial definable valuation ring. In this article, the author generalizes the construction of the group of infinitesimals to infinite fields of arbitrary but finite dp-rank, which allows him to prove in a sequel article [W. Johnson, Ann. Pure Appl. Logic 172, No. 6, Article ID 102949, 33 p. (2021; Zbl 07333020)] the existence of a non-trivial valuation ring.

Reviewer: Amador Martin-Pizarro (Freiburg)

MSC:
03C45 Classification theory, stability, and related concepts in model theory

Keywords:
dp-rank; NIP fields; Shelah’s conjecture

Full Text: DOI

References:
[21] Sinclair, Peter, Relationships between model theory and valuations on fields (2018), McMaster University, PhD thesis

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