Universal minimal flows of homeomorphism groups of high-dimensional manifolds are not metrizable. (English) Zbl 07335458

From the Introduction: “A central object in abstract topological dynamics is the universal minimal flow (UMF) of a topological group $G$, often denoted by $M(G)$. It is a canonical dynamical system associated to $G$ that is defined abstractly as the minimal $G$-flow that admits any minimal $G$-flow as a factor.” Recall that a $G$-flow is a continuous action of $G$ on a compact Hausdorff space; it is minimal if all orbits are dense. From the Abstract: “Answering a question of V. Uspenskij [Topol. Proc. 25(Spring), 301–308 (2000; Zbl 0996.22003)], we prove that if $X$ is a closed manifold of dimension 2 or higher or the Hilbert cube, then the universal minimal flow of Homeo$(X)$ is not metrizable. In dimension 3 or higher, we also show that the minimal Homeo$(X)$-flow consisting of all maximal, connected chains in $X$ has meager orbits.”

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MSC:

37C85 Dynamics induced by group actions other than $\mathbb{Z}$ and $\mathbb{R}$, and $\mathbb{C}$
37B05 Dynamical systems involving transformations and group actions with special properties (minimality, distality, proximality, expansivity, etc.)
57S05 Topological properties of groups of homeomorphisms or diffeomorphisms
22A05 Structure of general topological groups
22F05 General theory of group and pseudogroup actions
54H15 Transformation groups and semigroups (topological aspects)

Keywords:
universal minimal flow; abstract topological dynamics; minimal $G$-flow; homeomorphism groups; Polish group

Full Text: DOI arXiv

References:


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