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Madanha, Sesuai Yash
Average number of zeros of characters of finite groups. (English) Zbl 07335749
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Given a finite group G, let $\operatorname{anz}(G)$ be the average number of zeros in the rows of the character table. The main result in this paper shows that if $\operatorname{anz}(G) < 1$ then G is solvable. In order to prove this result, the author proves the following extendibility result: if N is a nonabelian minimal normal subgroup of a finite group G, then there exists a nonprincipal irreducible character of N that extends to an irreducible character χ of G that vanishes on at least two G-conjugacy classes. As could be expected, the proof of this result relies on the classification of finite simple groups.

Other related results are proved in this paper. For instance, if $\operatorname{anz}(G) < 1/2$ then G is supersolvable and if G has odd order and $\operatorname{anz}(G) < 1$ then G is supersolvable. It is also conjectured that this later result can be improved to: if G has odd order and $\operatorname{anz}(G) < 16/11$ then G is supersolvable.

Reviewer's remarks: The reviewer has proved that there are exactly 4 nonabelian odd order groups with $\operatorname{anz}(G) < 16/11$: they are the Frobenius groups of order $3 \cdot 7$, $3 \cdot 13$, $3 \cdot 19$ and $5 \cdot 11$. He has also obtained a classification of the finite groups with $\operatorname{anz}(G) < 1$ that is independent of the main result of this paper and does not use the above mentioned extendibility theorem. [*A. Moretó*, "Groups with a small average number of zeros in the character table", Preprint, arXiv:2106.01943].

Reviewer: Alexander Moretó (Valencia)

MSC:

20C15 Ordinary representations and characters
20D10 Finite solvable groups, theory of formations, Schunck classes, Fitting classes, π-length, ranks

Keywords:

zeros of characters; solvable groups; supersolvable groups; nilpotent groups; abelian groupsó

Full Text: DOI

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