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On superstability in the class of flat modules and perfect rings. (English) Zbl 07337076


Summary: We obtain a characterization of left perfect rings via superstability of the class of flat left modules with pure embeddings.

Theorem 0.1. For a ring \( R \) the following are equivalent.

1. \( R \) is left perfect.
2. The class of flat left \( R \)-modules with pure embeddings is superstable.
3. There exists a \( \lambda \geq (|R| + \aleph_0)^+ \) such that the class of flat left \( R \)-modules with pure embeddings has uniqueness of limit models of cardinality \( \lambda \).
4. Every limit model in the class of flat left \( R \)-modules with pure embeddings is \( \Sigma \)-cotorsion.

A key step in our argument is the study of limit models in the class of flat modules. We show that limit models with chains of long cofinality are cotorsion and that limit models are elementarily equivalent.

We obtain a new characterization via limit models of the rings characterized in Rothmaler [Comm. Algebra 30 (2002), pp. 3077-3085]. We show that in these rings the equivalence between left perfect rings and superstability can be refined. We show that the results for these rings can be used to extend Shelah result [1.2, Notre Dame J. Form. Log. 58 (2017), pp. 159-177] to classes of flat modules not axiomatizable in first-order logic.

MSC:
03C48 Abstract elementary classes and related topics
16B70 Applications of logic in associative algebras
03C45 Classification theory, stability, and related concepts in model theory
03C60 Model-theoretic algebra
13L05 Applications of logic to commutative algebra
16L30 Noncommutative local and semilocal rings, perfect rings
16D10 General module theory in associative algebras

Keywords:
superstability; perfect rings; limit models; cotorsion modules; flat modules; abstract elementary classes

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