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Glorious pairs of roots and abelian ideals of a Borel subalgebra. (English) [Zbl 07339569]

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Summary: Let \( \mathfrak{g} \) be a simple Lie algebra with a Borel subalgebra \( \mathfrak{b} \). Let \( \Delta^+ \) be the corresponding (po)set of positive roots and \( \theta \) the highest root. A pair \( \{ \eta, \eta' \} \subset \Delta^+ \) is said to be glorious, if \( \eta, \eta' \) are incomparable and \( \eta + \eta' = \theta \). Using the theory of abelian ideals of \( \mathfrak{b} \), we (1) establish a relationship of \( \eta, \eta' \) to certain abelian ideals associated with long simple roots, (2) provide a natural bijection between the glorious pairs and the pairs of adjacent long simple roots (i.e., some edges of the Dynkin diagram), and (3) point out a simple transform connecting two glorious pairs corresponding to the incident edges in the Dynkin diagram. In types \( \text{DE} \), we prove that if \( \{ \eta, \eta' \} \) corresponds to the edge through the branching node of the Dynkin diagram, then the meet \( \eta \wedge \eta' \) is the unique maximal non-commutative root. There is also an analogue of this property for all other types except type \( \text{A} \). As an application, we describe the minimal non-abelian ideals of \( \mathfrak{b} \).

MSC:

17B20 Simple, semisimple, reductive (super)algebras
17B22 Root systems
06A07 Combinatorics of partially ordered sets
20F55 Reflection and Coxeter groups (group-theoretic aspects)

Keywords:

root system; Borel subalgebra; abelian ideal; adjacent simple roots

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References:


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