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Covering numbers of commutative rings. (English) Zbl 07341175
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Summary: A cover of a unital, associative (not necessarily commutative) ring $R$ is a collection of proper subrings of $R$ whose set-theoretic union equals $R$. If such a cover exists, then the covering number $\sigma(R)$ of $R$ is the cardinality of a minimal cover, and a ring $R$ is called $\sigma$-elementary if $\sigma(R) < \sigma(R/I)$ for every nonzero two-sided ideal $I$ of $R$. In this paper, we show that if $R$ has a finite covering number, then the calculation of $\sigma(R)$ can be reduced to the case where $R$ is a finite ring of characteristic $p$ and the Jacobson radical $J$ of $R$ has nilpotency 2. Our main result is that if $R$ has a finite covering number and $R/J$ is commutative (even if $R$ itself is not), then either $\sigma(R) = \sigma(R/J)$, or $\sigma(R) = p^d + 1$ for some $d \geq 1$. As a byproduct, we classify all commutative $\sigma$-elementary rings with a finite covering number and characterize the integers that occur as the covering number of a commutative ring.

MSC:
16P10 Finite rings and finite-dimensional associative algebras
13M99 Finite commutative rings
05E16 Combinatorial aspects of groups and algebras

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subring cover; covering

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