E. Jespers and J. Okniński [Algebr. Represent. Theory 19, No. 1, 17–31 (2016; Zbl 1376.16021)] proved that semigroup algebras of subsemigroups of 2-nilpotent groups have well behaved prime spectrum. In particular, their prime ideals are completely prime (namely, the quotients with respect to which have no zero divisors) and therefore their classical Krull dimension is finite, bounded by the Hirsch length of the group. They also show that the situation for subsemigroups of 3-nilpotent groups is more complicated.

Of particular interest is the 2-generated free 3-nilpotent group

\[ G = \langle a, b, c, d, e | bc = acb, ab = dba, ac = eca, db = bd, dc = cd, eb = be, ec = ce \rangle \]

Note that \( G \) is generated as a group by \( b, c \). Consider its subsemigroup \( S \) generated by \( b, c \). For an arbitrary base field \( F \), the semigroup algebra \( F[S] \) is shown to have infinitely many non-completely prime homomorphic images which are monomial algebras, corresponding to the infinite periodic words \( (bc^i)^\infty \).

Thus, the corresponding homomorphic images of \( F[S] \) are PI (satisfy a polynomial identity) of linear growth. Jespers and Oknin\’ski raise the following open questions:

*Do there exist prime homomorphic images of \( F[S] \) that are not Goldie? In particular, could they be of the form \( F[S]/P \) for a prime ideal generated by elements from \( S \)? Can infinite chains of primes exist in \( S \)? Can \( F[S] \) have infinite classical Krull dimension?*

The main result partially answers these questions. The aim of the paper is to show that the only prime monomial homomorphic images of \( F[S] \) are just-infinite of at most quadratic growth. If they have linear growth, then they are precisely the quotients by the ideals discovered by Jespers and Oknin\’ski, and otherwise, they are primitive. Just-infinite algebras are infinite-dimensional algebras all of whose proper homomorphic images are finite-dimensional. The question of whether \( F[S] \) has prime monomial homomorphic images of precisely quadratic growth remains open.

For any superword \( W \) one can associate algebra

\[ A_W = F < x_1, \ldots, x_m > / u | u \text{ is not a factor of } W \].

The main theorem is:

**Theorem 4.1** If \( P \triangleleft F[S] \) is a prime monomial ideal, then \( F[S]/P \simeq A_W \) is just-infinite of at most quadratic growth. It is either PI of linear growth or primitive of quadratic growth.

The proof uses deep ideas from symbolic dynamics, and radical theory for monomial algebras. Namely, if monomial algebra \( A \) is just infinite (in other terminology almost simple) then \( A = A_W \) for some uniformly recurrent superword \( W \) [A. Ya. Belov et al., J. Math. Sci., New York 87, No. 3, 3463–3575 (1997; Zbl 0927.16018)].

Another important component of proof of the theorem 4.1 is Lemma 3.3. We want to attract attention to endomorphism iteration using in the proof of Theorem 4.1.

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**MSC:**

- 16S15 Finite generation, finite presentability, normal forms (diamond lemma, term-rewriting)
- 16N60 Prime and semiprime associative rings
- 20M18 Inverse semigroups

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