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De Morgan clones and four-valued logics. (English) Zbl 07345062
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A De Morgan clone is a clone over the four-element set $DM_4 := \{t, f, n, b\}$. This is the set of truth values of Benap-Dunn’s four-valued logic [N. D. Belnap jun., in: Mod. Uses of multiple-valued Logic, 5th int. Symp., Bloomington 1975, 5–37 (1977; Zbl 0424.03012)], known also as the logic of first order entailment [J. M. Dunn, Philos. Stud. 29, No. 3, 149–168 (1976; Zbl 1435.03043)]. The De Morgan algebra is the algebra $DM_4 := (DM_4, \lor, \land, t, f, \neg)$ where $\lor$ and $\land$ are the lattice operations w.r.t. the so called truth order given by $f \leq b \leq t, f \leq n \leq t$ and the operation $\neg$ is defined by $\neg t = f, \neg f = t, \neg n = n, \neg b = b$. $DMA$ is the clone of all term functions of $DM_4$.

In the paper, generating sets are found for the clones of all functions that preserve the subalgebras of $DM_4$, the automorphisms of $DM_4$, the truth ordering of $DM_4$, the information ordering of $DM_4$ and also for some combinations of these. Described are also clones that fail to preserve some of this structure. It is shown that $DMA$ has exactly three covers in the lattice of all four-valued clones, and described is the lattice of all De Morgan clones above $DMA$ which contain a discriminator function. Each of the latter clones determine an expansion of the Belnap-Dunn logic by additional connectives, and the author provides a classification of these clones by the metalogical properties of corresponding logics, namely, by their position in the Leibniz and Frege hierarchies.

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MSC:
- 03G27 Abstract algebraic logic
- 03C05 Equational classes, universal algebra in model theory
- 03G10 Logical aspects of lattices and related structures

Keywords: abstract algebraic logic; Belnap-Dunn logic; clone theory; four-valued logic

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