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On the birational geometry of spaces of complete forms. I: Collineations and quadrics.
(English) Zbl 07346497

Let $K$ be an algebraically closed field of characteristic zero. By Vainsencher, the space $X(n, m)$ of complete collineations $K^{n+1} \to K^{m+1}$ with $n \leq m$ can be obtained by the blow-ups of $\mathbb{P}^{(n+1)(m+1)-1}$ along the Segre variety $\sigma(\mathbb{P}^n \times \mathbb{P}^m)$ and strict transforms of its secant varieties, and the space $Q(n)$ of complete quadrics in $\mathbb{P}^n$ can be obtained by the blow-ups of $\mathbb{P}^{(n+2)\cdot 2-1}$ along the Veronese variety $v_2(\mathbb{P}^n)$ and strict transforms of its secant varieties. These algebraic varieties have been fundamental in enumerative geometry since 19th century, and they are extensively studied by many people such as Chasles, Giambelli, Hirst, Schubert, Segre, Semple, Tyrrell, Vainsencher, Kleiman, Laksov, Lascoux, Thorup, and Thaddeus. The purpose of the paper under review is to investigate the birational geometry of $X(n, m)$ and $Q(n)$ from the point of view of Mori theory. It is known that they are spherical varieties; in particular, they are Mori dream spaces. In this paper, the minimal sets of generators for the Cox rings and effective, nef, movable cones of $X(n, m)$ and $Q(n)$ are explicitly computed (Theorem 1.1 and Appendix), and the Mori chamber and stable base locus decompositions of the effective cones are given for some cases including $X(3, 3)$ and $X(2, m)$ (Theorems 1.2, 6.11, and 6.12).

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MSC:
14E30 Minimal model program (Mori theory, extremal rays)
14J45 Fano varieties
14N05 Projective techniques in algebraic geometry
14E07 Birational automorphisms, Cremona group and generalizations
14M27 Compactifications; symmetric and spherical varieties

Keywords:
spaces of complete collineations; spaces of complete quadrics; Mori dream spaces; Cox rings; cones of divisors; chamber decompositions

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