Summary: Our main objective is a further study of \( \mathcal{M} \)-factorizability in topological groups as defined in Zhang, Peng, He, Tkachenko (2020) [15]. We focus on topological-algebraic implications of \( \mathcal{M} \)-factorizability such as \( \tau \)-precompactness, pseudo-\( \tau \)-compactness and \( \tau \)-fineness. We also study products of topological groups and present necessary and sufficient conditions on the factors guaranteeing the \( \mathcal{M} \)-factorizability of products. Our main technical tool for this study is the new notion of \( \tau \)-fine topological group, where \( \tau > \omega \) is a cardinal. We prove the following dichotomy theorem: Every \( \mathcal{M} \)-factorizable topological group is either \( \mathbb{R} \)-factorizable or \( \omega_1 \)-fine.

Another dichotomy is established for the product of two groups. We prove that if the product \( G \times H \) of topological groups is \( \mathcal{M} \)-factorizable, then for every cardinal \( \tau > \omega \), either \( G \) is \( \tau \)-fine or \( H \) is pseudo-\( \tau \)-compact. We also show that the product \( G \times H \) is \( \mathcal{M} \)-factorizable provided \( G \) is a metrizable topological group with \( w(G) \leq \tau \) and \( H \) is a \( \tau \)-fine topological group with \( hl(H) \leq \tau \).

It is also proved that the product \( G \times H \) is \( \mathcal{M} \)-factorizable (\( \mathbb{R} \)-factorizable) whenever \( G \) is an arbitrary \( \mathcal{M} \)-factorizable (\( \mathbb{R} \)-factorizable) topological group and \( H \) is a locally compact separable metrizable topological group.

MSC:

- 22A05 Structure of general topological groups
- 54A25 Cardinality properties (cardinal functions and inequalities, discrete subsets)
- 54H11 Topological groups (topological aspects)
- 54A35 Consistency and independence results in general topology

Keywords:

\( \mathcal{M} \)-factorizability; \( \mathbb{R} \)-factorizability; feathered group; \( \tau \)-fine group; metrizable; \( \omega \)-narrow; \( \omega \)-balanced

Full Text: DOI

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