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M-factorizability of products and τ-fine topological groups.  (English) Zbl 07354969
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Summary: Our main objective is a further study of M-factorizability in topological groups as defined in Zhang, Peng, He, Tkachenko (2020) [15]. We focus on topological-algebraic implications of M-factorizability such as τ-precompactness, pseudo-τ-compactness and τ-fineness. We also study products of topological groups and present necessary and sufficient conditions on the factors guaranteeing the M-factorizability of products. Our main technical tool for this study is the new notion of τ-fine topological group, where τ > ω is a cardinal. We prove the following dichotomy theorem: Every M-factorizable topological group is either R-factorizable or ω₁-fine.

Another dichotomy is established for the product of two groups. We prove that if the product G × H of topological groups is M-factorizable, then for every cardinal τ > ω, either G is τ-fine or H is pseudo-τ-compact. We also show that the product G × H is M-factorizable provided G is a metrizable topological group with w(G) ≤ τ and H is a τ-fine topological group with hl(H) ≤ τ.

It is also proved that the product G × H is M-factorizable (R-factorizable) whenever G is an arbitrary M-factorizable (R-factorizable) topological group and H is a locally compact separable metrizable topological group.

MSC:
22A05 Structure of general topological groups
54A25 Cardinality properties (cardinal functions and inequalities, discrete subsets)
54H11 Topological groups (topological aspects)
54A35 Consistency and independence results in general topology

Keywords:
M-factorizability; R-factorizability; feathered group; τ-fine group; metrizable; ω-narrow; ω-balanced

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References:

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