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Factorizations of Schur functions. (English) Zbl 07355811

Summary: The Schur class, denoted by \( S(D) \), is the set of all functions analytic and bounded by one in modulus in the open unit disc \( D \) in the complex plane \( \mathbb{C} \), that is
\[
S(D) = \left\{ \varphi \in H^\infty(D) : \|\varphi\|_\infty := \sup_{z \in D} |\varphi(z)| \leq 1 \right\}.
\]
The elements of \( S(D) \) are called Schur functions. A classical result going back to I. Schur states: A function \( \varphi : D \to \mathbb{C} \) is in \( S(D) \) if and only if there exist a Hilbert space \( H \) and an isometry (known as colligation operator matrix or scattering operator matrix)
\[
V = \begin{bmatrix}
    a & B \\
    C & D
\end{bmatrix} : \mathbb{C} \oplus H \to \mathbb{C} \oplus H,
\]
such that \( \varphi \) admits a transfer function realization corresponding to \( V \), that is
\[
\varphi(z) = a + zB(I_H - zD)^{-1}C \quad (z \in D).
\]
An analogous statement holds true for Schur functions on the bidisc. On the other hand, Schur-Agler class functions on the unit polydisc in \( \mathbb{C}^n \) is a well-known “analogue” of Schur functions on \( D \). In this paper, we present algorithms to factorize Schur functions and Schur-Agler class functions in terms of colligation matrices. More precisely, we isolate checkable conditions on colligation matrices that ensure the existence of Schur (Schur-Agler class) factors of a Schur (Schur-Agler class) function and vice versa.

MSC:

32A10 Holomorphic functions of several complex variables
32A38 Algebras of holomorphic functions of several complex variables
32A70 Functional analysis techniques applied to functions of several complex variables
47A48 Operator colligations (= nodes), vessels, linear systems, characteristic functions, realizations, etc.
47A13 Several-variable operator theory (spectral, Fredholm, etc.)
46E15 Banach spaces of continuous, differentiable or analytic functions
93B15 Realizations from input-output data
15A23 Factorization of matrices
93C35 Multivariable systems, multidimensional control systems
32A38 Algebras of holomorphic functions of several complex variables
30H05 Spaces of bounded analytic functions of one complex variable
47N70 Applications of operator theory in systems, signals, circuits, and control theory
93B28 Operator-theoretic methods
94A12 Signal theory (characterization, reconstruction, filtering, etc.)

Keywords:

transfer functions; block operator matrices; colligation; scattering matrices; Schur class; Schur-Agler class; realization formulas

Full Text: DOI

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