An increasing homeomorphism $h : \mathbb{R} \to \mathbb{R}$ is quasisymmetric, $h \in QS(\mathbb{R})$, if $M^{-1} \leq (h(x + t) - h(x))/(h(x) - h(x - t)) \leq M$ for all $x \in \mathbb{R}$ and $t > 0$ and $h \in QS(\mathbb{R})$ belongs to the symmetric class $S(\mathbb{R})$ if $\lim_{t \to 0}(h(x + t) - h(x))/(h(x) - h(x - t)) = 1$ uniformly for all $x \in \mathbb{R}$.

Quasisymmetric and symmetric quasisymmetric mappings play a central role in the theory of plane quasiconformal mappings and Teichmüller theory. The main result of the paper states that there is $h \in S(\mathbb{R})$ but $h^{-1}$ is not symmetric. Earlier it has been known that $S(\mathbb{R})$ is not a topological group, see [M. A. Brakalova, Anal. Math. Phys. 8, No. 4, 541-549 (2018; Zbl 1414.30047)] and [H. Yun et al., Proc. Am. Math. Soc. 146, No. 10, 4255-4263 (2018; Zbl 1404.30031)]. The construction is explicit and rather complicated and, among other properties of quasisymmetric mappings, it is shown that $S(\mathbb{R})$ is not closed under composition. The authors also consider the class of strongly quasisymmetric homeomorphism $h$ in $\mathbb{R}$ which satisfy $|h(E)/h(I)| \leq c_1(|E|/|I|)^{c_2}$ whenever $I$ is an interval and $E \subset I$ is a measurable set, see [S. W. Semmes, Trans. Am. Math. Soc. 306, No. 1, 233-263 (1988; Zbl 0653.30008)], and show that this class is not preserved under taking the inverse and composition.

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30C62 Quasiconformal mappings in the complex plane
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