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Extension of Gyárfás-Sumner conjecture to digraphs. (English) Zbl 07356170

The dichromatic number of a digraph is the minimum number of colors needed to color its vertices in such a way that each color class induces an acyclic digraph [V. Neumann-Lara, J. Comb. Theory, Ser. B 33, 265–270 (1982; Zbl 0506.05031)]. On the other hand, Gyárfás and Sumner conjectured that: Given two graphs $F_1$ and $F_2$ the class of graphs with no induced $F_1$ or $F_2$ has bounded chromatic number if and only if one of $F_1$; $F_2$ is a complete graph and the other is a forest (see [A. Gyárfás, Zastosow. Mat. 19, No. 3–4, 413–441 (1987; Zbl 0718.05041)] and [D. P. Sumner, in: The theory and applications of graphs, 4th int. Conf., Kalamazoo/ Mich. 1980. 557–576 (1981; Zbl 0476.05037)]).

In this paper the authors look for possible extensions of the Gyárfás-Sumner conjecture. In particular, they conjecture a simple characterization of sets $F$ of three digraphs such that every digraph with sufficiently large dichromatic number must contain a member of $F$ as an induced subdigraph. Regarding this, the authors prove that oriented $K_4$-free graphs without a directed path of length 3 have bounded dichromatic number, where a bound of 414 is provided. They also show that an orientation of a complete multipartite graph with no directed triangle is 2-colorable. To prove these results the authors introduce the notion of “nice sets” that might be of independent interest.

Reviewer: Juan José Montellano Ballesteros (Ciudad de México)

MSC:
05C15 Coloring of graphs and hypergraphs
05C20 Directed graphs (digraphs), tournaments

Keywords:
digraphs; dichromatic number; Gyárfás-Sumner conjecture

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References:


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