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The center of the categorified ring of differential operators. (English) Zbl 07357366
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Summary: Let \mathcal{Y} be a derived algebraic stack satisfying some mild conditions. The purpose of this paper is three-fold. First, we introduce and study $\mathbb{H}(\mathcal{Y})$, a monoidal DG category that might be regarded as a categorification of the ring of differential operators on \mathcal{Y} . When $\mathcal{Y} = \text{LS}_G$ is the derived stack of G -local systems on a smooth projective curve, we expect $\mathbb{H}(\text{LS}_G)$ to act on both sides of the geometric Langlands correspondence, compatibly with the conjectural Langlands functor. Second, we construct a novel theory of D-modules on derived algebraic stacks. In contrast to usual D-modules, this new theory, to be denoted by \mathcal{D}^{der} , is sensitive to the derived structure. Third, we identify the Drinfeld center of $\mathbb{H}(\mathcal{Y})$ with $\mathcal{D}^{\text{der}}(L\mathcal{Y})$, the DG category of \mathcal{D}^{der} -modules on the loop stack $L\mathcal{Y} := \mathcal{Y} \times_{\mathcal{Y} \times \mathcal{Y}} \mathcal{Y}$.

MSC:

- [14D24](#) Geometric Langlands program (algebraic-geometric aspects)
- [14F08](#) Derived categories of sheaves, dg categories, and related constructions in algebraic geometry
- [14F10](#) Differentials and other special sheaves; D-modules; Bernstein-Sato ideals and polynomials
- [18F20](#) Presheaves and sheaves, stacks, descent conditions (category-theoretic aspects)

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Keywords:

derived algebraic geometry; coherent sheaves; formal completions; Hochschild cohomology; DG categories; Drinfeld center; D-modules

Full Text: [DOI](#) [arXiv](#)

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