Poppe, Harry

Ascoli-Arzela-theory based on continuous convergence in an (almost) non-Hausdorff setting 2. (English) Zbl 07357699

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Let $C(X,Y)$ be the space of continuous functions where X, Y are topological spaces. In [*H. Poppe*, Compactness in general function spaces. Berlin: VEB Deutscher Verlag der Wissenschaften (1974; Zbl 0291.54012), (3.24), (3.27), *R. Bartsh* et al., in: Categorical topology. Proceedings of the L'Aquila conference, August 31–September 4, 1994, L'Aquila, Italy. Dordrecht: Kluwer. 221–240 (1996; Zbl 0880.54015), Corollary 33 and *F. Mynard*, Real Anal. Exch. 38, No. 2, 431–444 (2013; Zbl 1295.54004)], necessary and sufficient conditions were given for H a subset $C(X,Y)$ to be relatively compact with respect to continuous convergence. In this article as a Corollary, is characterized the compactness of H.

Further the articles [*F. Mynard*, loc. cit. and *R. Bartsh* et al., loc. cit.] did not provide examples which show that the assumption of Y is Hausdorff can not be omitted; in the present article, such an example is given. Finally, it is well known that the notion of equicontinuity can be characterized using canonical maps used in duality theory. In the article [*G. Di Maio* et al., Mediterr. J. Math. 3, No. 2, 189–204 (2006; Zbl 1121.54030)] and the book [*S. Naimpally*, Proximity approach to problems in topology and analysis. München: Oldenbourg Verlag (2009; Zbl 1185.54001)], this approach was extended to include even continuity and also the topological equicontinuity of Royden. But the two Ascoli-Arzela Theorems [loc. cit., Theorems (13,15), (13,21)], based on this approach are not correct. In this article, the author gives a concrete example which shows that in general it does not hold: if f_n converges pointwise, and f_n is equicontinuinous, then f_n converges uniformly.

Reviewer: Ennis Rosas (Cumaná)

MSC:

54C35 Function spaces in general topology
54A20 Convergence in general topology (sequences, filters, limits, convergence spaces, nets, etc.)
54D30 Compactness
46A20 Duality theory for topological vector spaces
54C25 Embedding

Full Text: Link

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