Alzamel, A.; Wolfe, J. M.
Best multipoint local $L_p$ approximation. (English) [Zbl 0736.41030]

Let $I = [a, b]$ be an interval and $k, n$ positive integers with $k \leq n + 1$. Then $n + 1 = \ell \cdot k + r$ with integers $\ell, r$ such that $0 \leq r < k$. Let $M$ be an $n + 1$-dimensional extended Tchebycheff subspace of $C(I)$ and $x_i$ points with $a \leq x_1 < \cdots < x_k \leq b$. Let $f \in C(I)$ be fixed. Then for each fixed $1 \leq p \leq \infty$ and sufficiently small $h$ there is at least one $q_h \in M$ which is the best approximation to $f$ in $L^p(I_h)$, where $I_h$ is the union of intervals $[x_j, x_j + h]$. An element $q^* \in M$ is called a best $k$-point approximation to $f$ if there is a sequence $h_\nu \to 0$ such that the pertaining $q_{h_\nu} \to q^*$. The authors consider the problem of existence, uniqueness and characterization for such multipoint approximations. The main result is the following theorem: If $1 < p \leq \infty$ and $f \in C^{\ell+1}$ then $q_h \to q_0$ uniformly as $h \to 0$. The function $q_0 \in M$ is uniquely defined. It satisfies (*) $q_0^{(i)}(x_j) = f^{(i)}(x_j)$ for $j = 1, \ldots, k$ and $i = 0, \ldots, \ell - 1$. Further, $q_0$ minimizes the sum over $\|f^{(i)}(x_j) - q_0^{(i)}(x_j)\|^p$ over all $q \in M$ satisfying the interpolation conditions (*).

For $p = 1$ a similar result is obtained. But in this case there is in general no uniqueness and the $q_0$ is only a cluster point of the $q_h$.

Reviewer: R. Wegmann (München)

MSC:
41A50 Best approximation, Chebyshev systems
41A65 Abstract approximation theory (approximation in normed linear spaces and other abstract spaces)

Keywords:
best local $k$-point approximation

Full Text: DOI

References:

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