This monograph is an important contribution to the study of linear dynamical systems on Hilbert spaces, that is, the behavior of orbits of bounded linear operators acting on a Hilbert space. In this review $B(H)$ denotes the algebra of bounded linear operators on a complex separable infinite-dimensional Hilbert space $H$. Hypercyclic, chaotic and (upper) frequently hypercyclic operators are investigated in the memoir. In very general terms, the following type of questions are investigated: given two interesting dynamical properties, do there exist linear operators $T \in B(H)$ satisfying one of them but not the other? In particular, the following three questions are considered: (1) Are there operators which are frequently hypercyclic but not ergodic, in the sense that they do not admit an ergodic measure with full support? (2) Are there operators which are upper frequently hypercyclic but not frequently hypercyclic? and (3) Are there operators which are ergodic but do not admit a Gaussian ergodic measure with full support? The answer to the first two questions is known to be positive for operators on the Banach space $c_0$ by the work of F. Bayart and I. Z. Rousu [Ergodic Theory Dyn. Syst. 35, No. 3, 691–709 (2015; Zbl 1355.37035)] and the first two authors [Adv. Math. 265, 371–427 (2014; Zbl 1303.47013)]. Two different strategies are consider by the authors to attack these problems: firstly, try to determine which properties are generic and which ones are not, in a Baire category sense, fixing an appropriate topological setting with the strong operator topology and the strong* operator topology, which are both separable and completely metrizable (thus Polish spaces) when restricted to closed balls. Secondly, when it is unclear how a Baire category approach could work, one tries to construct explicit examples of operators satisfying (or not) one of the properties. To do this, the authors consider the kind of operators constructed by the third author in [Trans. Am. Math. Soc. 369, No. 7, 4977–4994 (2017; Zbl 1454.47014)]. A general scheme is presented allowing them to produce operators on $H$ which are chaotic and frequently hypercyclic but not ergodic, as well as operators on $H$ which are chaotic and upper frequently hypercyclic but not frequently hypercyclic. These are the first examples of this type of operators acting on a Hilbert space. Moreover it is shown that there are operators $T \in B(H)$ which are chaotic and topologically mixing but not upper frequently hypercyclic. The proofs rely on new criteria for frequent hypercyclicity and upper frequent hypercyclicity, stronger than the known ones, in which the periodic points play a central role. The problem to determine the exact descriptive complexity of a given class of operators is also treated, in particular for chaotic operators and for topologically mixing operators. A partial result for upper frequently hypercyclic operators, whose proof relies on our general scheme for constructing operators with special properties, is also included. We briefly describe the content of each chapter. Recall that a subset of a Polish space $X$ is said to be meager in $X$ if it can be covered by countably many closed sets with empty interior, and comeager if its complement is meager. In this sense, a property of elements of $X$ is called typical if the set of all $x \in X$ satisfying it is comeager in $X$, and that a property is atypical if its negation is typical. In Chapter 2, a few basic facts about the strong operator topology and the strong* operator topology are recalled, and then “typicality” results in the closed balls of $H$ and in the set of hypercyclic vectors contained in a ball are given. Chapter 3 discusses the descriptive complexity of some of the families of operators under study. In Chapter 4 the authors investigate ergodicity properties of upper triangular operators with coefficients of modulus 1 on the diagonal, with respect to the strong operator topology. In Chapter 5 the criteria for an operator $T \in B(H)$ to be upper frequently hypercyclic or frequently hypercyclic mentioned above are presented. They are stated in the general setting of Banach spaces and could be useful in other contexts. Chapter 6 is the most difficult part of this work. Here a general machinery for producing hypercyclic operators with special properties is presented. The operators constructed here depend on a number of parameters, and the authors are able to determine in a rather precise way how the parameters influence on upper frequent or frequent hypercyclicity, ergodicity or topological mixing. This machinery is used to produce the examples in Chapter 7. A list of fifteen interesting open questions is included in the last Chapter 8. There is an updated list of references.

Reviewer: José Bonet (Valencia)
MSC:

47-02 Research exposition (monographs, survey articles) pertaining to operator theory

47A16 Cyclic vectors, hypercyclic and chaotic operators

47A35 Ergodic theory of linear operators

37A05 Dynamical aspects of measure-preserving transformations

54E52 Baire category, Baire spaces

54H05 Descriptive set theory (topological aspects of Borel, analytic, projective, etc. sets)

Keywords:
linear dynamical systems; Hilbert spaces; Baire category; frequent and \(U\)-frequent hypercyclicity; ergodicity; chaos; topological mixing

Full Text: DOI

References:


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