A field $K$ is immediately algebraically closed (IAC) if, for every nontrivial valuation $v$ on $K$, the residue field $K^v$ is algebraically closed and the value group $vK$ is divisible. The field $K$ is valuationally algebraically closed (VAC) if, for every nontrivial valuation $v$ on $\bar{\bar{K}}$, its algebraic closure, $\bar{K}$ is dense in $\bar{\bar{K}}$ with respect to $v$. Examples of VAC fields include, separably closed fields (proposition 4.9), pseudo-algebraically closed fields and super-rosy fields of positive characteristic. Conversely, it follows from Krasner’s Lemma that a VAC field supporting a non-trivial henselian valuation is separably closed (Proposition 2.10).

It is clear from the definition that a VAC field is also IAC, but the reverse implication is not true, as shown by a couple of examples in Section 3 of the paper. In Section 5 it is shown, however, that when considering IAC as a property of the theory of $K$, rather than of a single field $K$, it does imply VAC.

In a different direction it is shown (Section 4) that IAC implies VAC if $K$ is Artin-Schreier closed (in positive characteristic) or has a divisible multiplicative group (in the characteristic 0). These results are based on the fact (Proposition 4.3) that if $K$ is IAC then to show that it is VAC it suffices to check that for any rank 1-valuation $v$ on $\bar{K}$ the completion of $(K, v)$ is algebraically closed.

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References:


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