Let $G_n = a_1 \alpha_1^n + \ldots + a_t \alpha_t^n$ be the $n$-th term of a non-degenerate linear recurrence sequence in the power sum representation. In the number field case, if $\max_{1 \leq j \leq t} |\alpha_j| \geq 1$, then

$$|G_n| \geq (\max_{1 \leq j \leq t} |\alpha_j|)^{n(1-\varepsilon)}$$

is well-known, for sufficiently large $n$.

In the present paper the authors prove a function field analogue of this theorem, in case of function fields in one variable of characteristic zero. The main result is the following:

Let $(G_n)_{n=0}^\infty$ be a nondegenerate linear recurrence sequence taking values in the function field $K$ with power sum representation $G_n = a_1 \alpha_1^n + \ldots + a_t \alpha_t^n$. Let $L = K(\alpha_1, \ldots, \alpha_t)$ be the splitting field of the characteristic polynomial of that sequence and let $\mu$ be a valuation on $L$. Then there is an effectively computable constant $C$, independent of $n$, such that, for every sufficiently large $n$, the inequality

$$\mu(G_n) \leq C + n \cdot \min_{1 \leq j \leq t} \mu(\alpha_j)$$

holds.


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MSC:

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