A subgroup $\sigma = \{\sigma_i | i \in I\}$ be a partition of the set of all primes $P$. The group $G$ is said to be: $\sigma$-primary if $G$ is a $\sigma_i$-group for some $i = i(G)$; $\sigma$-nilpotent if $G = G_1 \times \cdots \times G_n$ for some $\sigma$-primary groups $G_1, \ldots, G_n$; $\sigma$-soluble if every chief factor of $G$ is $\sigma$-primary; $\sigma$-full if $G$ possesses a Hall $\sigma_i$-subgroup for all $i$ such that $\sigma_i \cap \pi(G) \neq \emptyset$.

A subgroup $A$ of $G$ is said to be $\sigma$-permutable in $G$ provided $G$ is $\sigma$-full and $A$ permutes with every Hall $\sigma_i$-subgroup $H$ of $G$, that is, $AH = HA$ for all $i$; $G$ is said to be a $P\sigma T$-group if $\sigma$-permutability is a transitive relation in $G$, that is, if $K$ is a $\sigma$-permutable subgroup of $H$ and $H$ is a $\sigma$-permutable subgroup of $G$, then $K$ is a $\sigma$-permutable subgroup of $G$.

Let $\mathcal{F}$ be a class of group. Then a set $\Sigma$ of subgroups of $G$ is called a $G$-covering subgroup system for the class $\mathcal{F}$ if $G \in \mathcal{F}$ whenever $\Sigma \subseteq \mathcal{F}$.

We prove that: If a set of subgroups $\Sigma$ of $G$ contains at least one supplement to each maximal subgroup of every Sylow subgroup of $G$, then $\Sigma$ is a $G$-covering subgroup system for the classes of all $\sigma$-soluble groups, all $\sigma$-nilpotent groups, and all $\sigma$-soluble $P\sigma T$-groups. This result gives positive answers to Questions 19.87 and 19.88 in the Kourovka Notebook and, also, allows us to obtain the following characterization of $\sigma$-soluble $P\sigma T$-groups: $G$ is a $\sigma$-soluble $P\sigma T$-group if and only if each maximal subgroup of every Sylow subgroup of $G$ has a supplement $T$ in $G$ such that $T$ is a $\sigma$-soluble $P\sigma T$-group.

MSC:

20D10 Finite soluble groups, theory of formations, Schunck classes, Fitting classes, $\pi$-length, ranks

Keywords:
finite group; $G$-covering subgroup system; $\sigma$-soluble group; $\sigma$-nilpotent group; $P\sigma T$-group

Full Text: DOI

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