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Existence and multiplicity results for a class of coupled quasilinear elliptic systems of gradient type. (English) [Zbl 07374864]


Summary: The aim of this paper is investigating the existence of one or more weak solutions of the coupled quasilinear elliptic system of gradient type

\[
\begin{cases}
- \text{div}(A(x, u)|\nabla u|^{p_1-2}\nabla u) + \frac{1}{p_1} A_u(x, u)|\nabla u|^{p_1} = G_u(x, u, v) & \text{in } \Omega, \\
- \text{div}(B(x, v)|\nabla v|^{p_2-2}\nabla v) + \frac{1}{p_2} B_v(x, v)|\nabla v|^{p_2} = G_v(x, u, v) & \text{in } \Omega, \\
u = v = 0 & \text{on } \partial \Omega,
\end{cases}
\]

where \( \Omega \subset \mathbb{R}^N \) is an open bounded domain, \( p_1, p_2 > 1 \) and \( A(x, u), B(x, v) \) are \( C^1 \)-Carathéodory functions on \( \Omega \times \mathbb{R} \) with partial derivatives \( A_u(x, u) \), respectively \( B_v(x, v) \), while \( G_u(x, u, v), G_v(x, u, v) \) are given Carathéodory maps defined on \( \Omega \times \mathbb{R} \times \mathbb{R} \) which are partial derivatives of a function \( G(x, u, v) \). We prove that, even if the coefficients make the variational approach more difficult, under suitable hypotheses of functional \( J \), related to problem (P), admits at least one critical point in the “right” Banach space \( X \). Moreover, if \( J \) is even, then (P) has infinitely many weak bounded solutions. The proof, which exploits the interaction between different norms, is based on a weak version of the Cerami-Palais-Smale condition, a “good” decomposition of the Banach space \( X \) and suitable generalizations of the Ambrosetti-Rabinowitz Mountain Pass Theorems.

MSC:

35J57 Boundary value problems for second-order elliptic systems
35J92 Quasilinear elliptic equations with p-Laplacian
35A01 Existence problems for PDEs: global existence, local existence, non-existence
35J50 Variational methods for elliptic systems
58E05 Abstract critical point theory (Morse theory, Lyusternik-Shnirel’man theory, etc.) in infinite-dimensional spaces

Keywords:
coupled quasilinear elliptic system; existence and multiplicity

Full Text: DOI

References:


