The paper under review considers graphs with no $K_t$-minor. The main result proved is that given $t \geq 1$, there is a constant $c_t$ such that for any $K_t$-free graph $G$ and every set of balls in $G$, the minimum size of a set of vertices intersecting all the balls is at most $c_t$ times the size of a largest collection of disjoint balls. Here as usual the ball of radius $r$ centred at a vertex $v$ in a graph is the set of all vertices at graph distance at most $r$ from $v$ in the graph. In the language of hypergraph theory, this states that the transversal number of the hypergraph $H$ with vertex set $V(G)$ and hyperedges the set $S$ of balls has transversal number (smallest size of a set of vertices intersecting all edges), $\tau(H)$ at most $c_t$ times the matching number $\mu(H)$. (Note that the inequality $\tau(H) \geq \nu(H)$ is immediate). It is important to understand that $c_t$ does not depend on the radii of the balls. Of course the result applies in particular to planar graphs as planarity implies no $K_5$-minor. The proofs use the Erdős-Pósa property of the ball hypergraphs and results on their Vapnik-Chervonenkis dimension. The proof of the main result is constructive and could be turned into an efficient algorithm to find a transversal.

Reviewer: David B. Penman (Colchester)

MSC:

05C83 Graph minors
05C10 Planar graphs; geometric and topological aspects of graph theory
05C12 Distance in graphs
05C65 Hypergraphs
05C69 Vertex subsets with special properties (dominating sets, independent sets, cliques, etc.)

Keywords:

packing; covering

Full Text: DOI arXiv

References:


[21] Zbl 0880.05050


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