If $G$ is a topological group, the space of group homomorphisms $\text{Hom}(\mathbb{Z}^n, G)$ can be regarded as a topological subspace of the cartesian product $G^n$. This paper considers the homological stability of the space $\text{Hom}(\mathbb{Z}^n, G)$ and related spaces such as the space of representations $\text{Rep}(\mathbb{Z}^n, G) = \text{Hom}(\mathbb{Z}^n, G)/G$ and the classifying space $B_{\text{com}}(G)$ of transitionally commutative $G$-bundles, where $G$ is a Lie group (see [A. Adem et al., Math. Proc. Camb. Philos. Soc. 152, No. 1, 91–114 (2012; Zbl 1250.57003)]. As their main tools, the authors use the work on rational homological stability as developed in [T. Church and B. Farb, Adv. Math. 245, 250–314 (2013; Zbl 1300.20051) and in [J. C. H. Wilson, J. Algebra 420, 269–332 (2014; Zbl 1344.20023)].

One of their main results is described as follows. Let $G_r$ be one of $\text{SU}(r), \text{U}(r), \text{SO}(2r+1), \text{Sp}(r)$, or $\text{SO}(2r)$, and fix $k, n \geq 0$. Then the standard inclusion $G_r \to G_{r+1}$ induces isomorphisms

$$H_k(\text{Hom}(\mathbb{Z}^n, G_r)_1; \mathbb{Q}) \to H_k(\text{Hom}(\mathbb{Z}^n, G_{r+1})_1; \mathbb{Q})$$

$$H_k(B_{\text{com}}(G_r)_1; \mathbb{Q}) \to H_k(B_{\text{com}}(G_{r+1})_1; \mathbb{Q})$$

for $r - \lfloor \sqrt{r} \rfloor \geq k$; and

$$H_k(\text{Rep}(\mathbb{Z}^n, G_r)_1; \mathbb{Q}) \to H_k(\text{Rep}(\mathbb{Z}^n, G_{r+1})_1; \mathbb{Q})$$

for $\geq k$.

Here the sub-index 1 stands for the connected component of the trivial homomorphism.

Using different tools, the rational homological stability of $\text{Hom}(\mathbb{Z}^n, G_r)_1$ and the best possible stable range for classical groups different from $\text{SO}(r)$ have been obtained in [D. Kishimoto and M. Takeda, Adv. Math. 386, Article ID 107809, 43 p. (2021; Zbl 07367645)].

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MSC:

- 57T10 Homology and cohomology of Lie groups
- 20C30 Representations of finite symmetric groups
- 57M07 Topological methods in group theory
- 22E99 Lie groups

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