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Sub-posets in \( \omega^\omega \) and the strong Pytkeev* property. (English) [Zbl 07387388]
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Summary: Tukey order is used to compare the cofinal complexity of partially order sets (posets). We prove that there is a \( 2^c \)-sized collection of sub-posets in \( 2^\omega \) which forms an antichain in the sense of Tukey ordering. Using the fact that any boundedly-complete sub-poset of \( \omega^\omega \) is a Tukey quotient of \( \omega^\omega \), we answer two open questions published in [14].

The relation between \( P \)-base and strong Pytkeev* property is investigated. Let \( P \) be a poset equipped with a second-countable topology in which every convergent sequence is bounded. Then we prove that any topological space with a \( P \)-base has the strong Pytkeev* property. Furthermore, we prove that every uncountably-dimensional locally convex space (lcs) with a \( P \)-base contains an infinite-dimensional metrizable compact subspace. Examples in function spaces are given.

MSC:
54D70 Base properties of topological spaces
06A06 Partial orders, general
46B50 Compactness in Banach (or normed) spaces

Keywords:
Tukey order; strong Pytkeev* property; \( \omega^\omega \)-base; \( K(M) \)-base; \( P \)-base; locally convex space (lcs); posets; function spaces

Full Text: DOI

References:
[10] Feng, Z., P-bases and topological groups