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Iwasawa theory for p-adic representations. (English) Zbl 0739.11045

Algebraic number theory - in honor of K. Iwasawa, Proc. Workshop Iwasawa Theory Spec. Values L-Funct., Berkeley/CA (USA) 1987, Adv. Stud. Pure Math. 17, 97-137 (1989).

[For the entire collection see Zbl 0721.00006.]

The author proposes an Iwasawa theoretic realization of motives [for a more geometric approach, see *P. Schneider's* "Motivic Iwasawa theory", Adv. Stud. Pure Math. 17, 421-456 (1989; Zbl 0741.11042)] and attached p-adic L-functions, by what he calls a "compatible system" of ℓ-adic representations $V = \{V_\ell\}$ over $\mathbb{Q}$. Since the precise definitions are technical and lengthy, we'll only sketch (and a little bit approximatively) the main lines: V_ℓ is a $\mathbb{Q}_\ell$- vector space of finite dimension $d = d_V$, on which $G_{\mathbb{Q}}$ (and also $G_{\mathbb{Q}_p}$, for any prime q) acts. Take a prime p which is "ordinary" for V, in the sense that V_p admits a filtration of $\mathbb{Q}_p$-subspaces involving some inertia conditions. Choose a $G_{\mathbb{Q}}$-lattices T_p contained in V_p, called the "Tate module". Let $A_p = V_p/T_p$ and consider the "Selmer group": $S_{A_p}(\mathbb{Q}_\infty) = \{\sigma \in H^1(\mathbb{Q}_\infty, A_p); \sigma$ locally trivial at all places of $\mathbb{Q}_\infty\}$ (here, "locally trivial" means with respect to inertia). The main object is the Λ-module $\hat{S}_{A_p}(\mathbb{Q}_\infty)$, where $(\hat{\ })$ denotes the Pontryagin dual and Λ the usual Iwasawa algebra. In the classical (= cyclotomic and elliptic) cases, one recovers the usual Iwasawa modules. Here, $\hat{S}_{A_p}(\mathbb{Q}_\infty)$ is expected to give rise to p-adic L-functions in the following way: For every prime q, for $\ell \neq q$, denote $\mathrm{Frob}(p)$ the arithmetic Frobenius and define $E_q(T) = \det(I - \mathrm{Frob}(q)T \mid (V_\ell)_{unr})$. With certain assumptions on $E_q(T)$, and for $\mathrm{Re}(s) \gg 0$, define the complex L-function: $L_V(s) = \prod_q E_q(q^{-s})^{-1}$. There should be a functional equation relating $L_V(2-s)$ and $L_{V^*}(s)$ (where $()^*$ denotes duals with values in $\mathbb{Q}_\ell(1)$), via some factor $\Gamma_V(s)$. Call r_V the order of the pole for $\Gamma_V(s)$ at $s = 1$.

Conjecture 1: $\hat{S}_{A_p}(\mathbb{Q}_\infty)$ has Λ-rank equal to r_V.

In the particular case where $r_V = r_{V^*} = 0$, there should exist a p-adic L-function $L_p(\varphi, V)$, for $\varphi \in \hat{\Gamma}$, the properties of which imply the existence of a certain $\lambda_V \in \Lambda$, conjecturally described by

Conjecture 2: $\hat{S}_{A_p}(\mathbb{Q}_\infty)$ has characteristic ideal (λ_V).

In the usual (= cyclotomic and elliptic) cases, the above conjectures are classical results and conjectures of Iwasawa theory. Here, using Poitou- Tate theorems on Galois cohomology, the author proves a weak version of conjecture 1, and also a reflection theorem relating $\hat{S}_{V_p/T_p}(\mathbb{Q}_\infty)$ and a twisted version of $\hat{S}_{V_p^*/T_p^*}(\mathbb{Q}_\infty)$.

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11S25 Galois cohomology
11S40 Zeta functions and L-functions

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