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On the geometry of nearly orthogonal lattices. (English) Zbl 07390136

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The paper investigates well rounded nearly orthogonal lattices. A lattice is called well rounded if its minimal vectors span the space. The nearly orthogonal lattices are those that have a basis for which the angle between any ray spanned by a basis vector and any subspace spanned by a subset of the basis is at most $\pi/3$. The main results of the paper are well exposed in the introduction: The authors show that in dimension $n \geq 3$ well rounded nearly orthogonal lattices never provide local maxima of the density function and the only local minimum is the $\mathbb{Z}^n$-lattice. Nevertheless there are many eutactic or even strongly eutactic lattices to be found in the set of nearly orthogonal lattices. Explicit examples for nearly orthogonal lattices with many short vectors are given together with the sharp bound $3n$ on their cardinality. Another useful proven fact is that any collection of linearly independent unit vectors, whose inner products are bounded by some explicit constant $c_n$, span a nearly orthogonal lattice

Reviewer: Gabriele Nebe (Aachen)

MSC:

11H06 Lattices and convex bodies (number-theoretic aspects)
11H31 Lattice packing and covering (number-theoretic aspects)
52C17 Packing and covering in $n$ dimensions (aspects of discrete geometry)
42C15 General harmonic expansions, frames

Keywords:

nearly orthogonal lattices; well-rounded lattices; eutactic lattices; perfect lattices; sphere packing; coherence; frames

Full Text: DOI

References:


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