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On atomic ideals in some factor rings of $C(X, Z)$. (English) Zbl 07396222

Summary: A nonzero $R$-module $M$ is atomic if for each two nonzero elements $a, b$ in $M$, both cyclic submodules $Ra$ and $Rb$ have nonzero isomorphic submodules. In this article it is shown that for an infinite $P$-space $X$, the factor rings $C(X, Z)/C_F(X, Z)$ and $C_c(X)/C_F(X)$ have no atomic ideals. This fact generalizes a result published in paper by A. Mozaffarikhah et al. [J. Algebra Appl. 19, No. 4, Article ID 2050078, 22 p. (2020; Zbl 1442.16002)], which says that for an infinite set $X$, the factor ring $Z^X/Z^{(X)}$ has no atomic ideal. Another result is that for each infinite $P$-space $X$, the socle of the factor ring $C_c(X)/C_F(X)$ is always equal to zero. Also, zero-dimensional spaces $X$ are characterized for which $C^F(X, Z)/C_F(X, Z)$ have atomic ideals.

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