Akhadkulov, H.; Ying, T. Y.; Saaban, A. B.; Noorani, M. S.; Ibrahim, H.
Notes on Krasnoselskii-type fixed-point theorems and their application to fractional hybrid
differential problems. (English) [Zbl 07396555]
Fixed Point Theory 22, No. 2, 465-480 (2021)

Summary: In this paper we prove a new version of Krasnoselskii’s fixed-point theorem under a $(\psi, \theta, \varphi)$-weak contraction condition. The theoretical result is applied to prove the existence of a solution of the following fractional hybrid differential equation involving the Riemann-Liouville differential and integral operators orders of $0 < \alpha < 1$ and $\beta > 0$:

\[
\begin{aligned}
D^\alpha [x(t) - f(t, x(t))] &= g(t, x(t), I^\beta (x(t))), \text{ a.e. } t \in J, \beta > 0, \\
x(t_0) &= x_0,
\end{aligned}
\]

where $D^\alpha$ is the Riemann-Liouville fractional derivative order of $\alpha$, $I^\beta$ is Riemann-Liouville fractional integral operator order of $\beta > 0$, $J = [t_0, t_0 + a]$, for some fixed $t_0 \in \mathbb{R}, a > 0$ and the functions $f : J \times \mathbb{R} \to \mathbb{R}$ and $g : J \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ satisfy certain conditions. An example is also furnished to illustrate the hypotheses and the abstract result of this paper.

MSC:
47-XX Operator theory
26A33 Fractional derivatives and integrals
34A08 Fractional ordinary differential equations
34A12 Initial value problems, existence, uniqueness, continuous dependence and continuation of solutions to ordinary differential equations
47H07 Monotone and positive operators on ordered Banach spaces or other ordered topological vector spaces
47H10 Fixed-point theorems

Keywords:
fixed-point theorem; Riemann-Liouville fractional derivative; hybrid initial value problem

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