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Convolutions of sets with bounded VC-dimension are uniformly continuous. (English)

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Summary: We study a notion of VC-dimension for subsets of groups, defining this for a set $A$ to be the VC-dimension of the family $\{(xA) \cap A : x \in A \cdot A^{-1}\}$. We show that if a finite subset $A$ of an abelian group has bounded VC-dimension, then the convolution $1_A \ast 1_{-A}$ is Bohr uniformly continuous, in a quantitatively strong sense. This generalises and strengthens a version of the stable arithmetic regularity lemma of Terry and Wolf in various ways. In particular, it directly implies that the polynomial Bogolyubov-Ruzsa conjecture – a strong version of the polynomial Freiman-Ruzsa conjecture – holds for sets with bounded VC-dimension. We also prove some results in the non-abelian setting.

In some sense, this gives a structure theorem for translation-closed set systems with bounded (classical) VC-dimension: if a VC-bounded family of subsets of an abelian group is closed under translation, then each member has a simple description in terms of Bohr sets, up to a small error.

MSC:

11B30 Arithmetic combinatorics; higher degree uniformity
43A60 Almost periodic functions on groups and semigroups and their generalizations (recurrent functions, distal functions, etc.); almost automorphic functions

Keywords:

VC dimension; uniform continuity; convolutions; regularity lemma

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