Summary: We study the topology of small covers from their fundamental groups. We find a way to obtain explicit presentations of the fundamental group of a small cover. Then we use these presentations to study the relations between the fundamental groups of a small cover and its facial submanifolds. In particular, we can determine when a facial submanifold of a small cover is $\pi_1$-injective in terms of some purely combinatorial data on the underlying simple polytope. In addition, we find that any three-dimensional small cover has an embedded non-simply connected $\pi_1$-injective surface. Using this result and some results of Schoen and Yau [25], we characterize all the three-dimensional small covers that admit Riemannian metrics with nonnegative scalar curvature.

MSC:

57S12 Toric topology
53Cxx Global differential geometry
57Kxx Low-dimensional topology in specific dimensions

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