Summary: Let $G$ be a nonempty closed subset of a Banach space $X$. Let $\mathcal{B}(X)$ be the family of nonempty bounded closed subsets of $X$ endowed with the Hausdorff distance and $\mathcal{B}_G(X) = \{A \in \mathcal{B}(X) : A \cap G \neq \emptyset\}$, where the closure is taken in the metric space $(\mathcal{B}(X), H)$. For $x \in X$ and $F \in \mathcal{B}_G(X)$, we denote the nearest point problem $\inf\{|x - g| : g \in G\}$ by $\min(x, G)$ and the mutually nearest point problem $\inf\{|f - g| : f \in F, g \in G\}$ by $\min(F, G)$. In this paper, parallel to well-posedness of the problems $\min(x, G)$ and $\min(F, G)$ which are defined by De Blasi et al., we further introduce the weak well-posedness of the problems $\min(x, G)$ and $\min(F, G)$. Under the assumption that the Banach space $X$ has some geometric properties, we prove a series of results on weak well-posedness of $\min(x, G)$ and $\min(F, G)$. We also give two sufficient conditions such that two classes of subsets of $X$ are almost Chebyshev sets.

MSC:
46B20 Geometry and structure of normed linear spaces
41A65 Abstract approximation theory (approximation in normed linear spaces and other abstract spaces)
54E52 Baire category, Baire spaces

Keywords:
nearest point problem; mutually nearest point problem; weak well-posedness; relatively boundedly weakly compact set; strict convexity; dense $G_δ$-subset

Full Text: DOI

References:


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