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On the bounds for the generalized distance energy of graphs.  (Chinese. English summary)

Summary: For simple connected graph $G$, the generalized distance matrix $D_\alpha(G)$ is a convex combination of $Tr(G)$ and $D(G)$, and is defined as $D_\alpha(G) = \alpha Tr(G) + (1 - \alpha)D(G)$, for $0 \leq \alpha \leq 1$. If $\partial_1 \geq \partial_2 \geq \cdots \geq \partial_n$ are the eigenvalues of $D_\alpha(G)$, we define the generalized distance energy of $G$ as $E^{D_\alpha}(G) = \sum_{i=1}^{n} |\partial_i - \frac{2\alpha W(G)}{n}|$, where $W(G)$ is the Wiener index of $G$. In this paper, we first discuss some upper and lower bounds on the generalized distance energy $E^{D_\alpha}(G)$ of graphs for $\alpha \in (0, \frac{1}{2}]$, and study the case when $\alpha \in (\frac{1}{2}, 1]$, so as to expand the range of $\alpha$ in known bounds. Secondly, under the condition of preserving the main characteristics of distance energy, we obtain some upper and lower bounds on the generalized distance energy $E^{D_\alpha}(G)$ of graphs. Finally, we get the generalized distance energy of complete $k$-partite graphs.

MSC:
05C12 Distance in graphs
05C50 Graphs and linear algebra (matrices, eigenvalues, etc.)

Keywords:
distance Laplacian matrix; distance signless Laplacian matrix; generalized distance matrix; energy