Deng, Naijuan

**Number of solutions to** \((an)^x + (bn)^y = (cn)^z\). (Chinese. English summary) [Zbl 07404464]


Summary: Let \(a, b, c, n\) be positive integers such that \(\gcd(a, b) = \gcd(a, c) = \gcd(b, c) = \gcd(n, bc) = 1\), \(\min\{a, b, c, n\} > 1\). In this paper, we prove that there are at most two solutions in positive integers \((x, y, z)\) to the equation \((an)^x + (bn)^y = (cn)^z\) when \(c > \max\{a, c\}\). What’s more, there is at most one solution in positive integers \((x, y, z)\) to the equation \((an)^x + (bn)^y = (cn)^z\), when \(c < \max\{a, b\}\), \(P(n) \nmid a\) or \(a < c < b\), \(c \leq \frac{a+b}{2}\), \(P(n) \nmid a\).

**MSC:**

11D61 Exponential Diophantine equations

**Keywords:**

Diophantine equation; positive integer solution; number of solution