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Generalized commutators and the Moore-Penrose inverse. (English) Zbl 07404611
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Summary: This work studies the kernel of a linear operator associated with the generalized k-fold commutator. Given a set \( A = \{A_1, \ldots, A_k\} \) of real \( n \times n \) matrices, the commutator is denoted by
\[ [A_1| \ldots | A_k]. \]
For a fixed set of matrices \( A \) we introduce a multilinear skew-symmetric linear operator
\[ T_A(X) = T(A_1, \ldots, A_k)[X] = [A_1| \ldots | A_k|X]. \]
For fixed \( n \) and \( k \geq 2n-1 \), \( T_A \equiv 0 \) by the Amitsur-Levitski Theorem [2], which motivated this work. The matrix representation \( M \) of the linear transformation \( T \) is called the k-commutator matrix. \( M \) has interesting properties, e.g., it is a commutator; for \( k \) odd, there is a permutation of the rows of \( M \) that makes it skew-symmetric. For both \( k \) and \( n \) odd, a provocative matrix \( S \) appears in the kernel of \( T \). By using the Moore-Penrose inverse and introducing a conjecture about the rank of \( M \), the entries of \( S \) are shown to be quotients of polynomials in the entries of the matrices in \( A \). One case of the conjecture has been recently proven by Brassil. The Moore-Penrose inverse provides a full rank decomposition of \( M \).

MSC:
47B47 Commutators, derivations, elementary operators, etc.
47A05 General (adjoints, conjugates, products, inverses, domains, ranges, etc.)

Keywords:
generalized commutator; Amitsur-Levitski theorem; Moore-Penrose inverse

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References: