Bychkov, Boris Sergeevich; Shabat, Georgii Borisovich

On generalizations of Chebyshev polynomials and Catalan numbers. (Russian. English translation) Zbl 07407664

Summary: We provide possible directions of generalizations of earlier found relations between the Chebyshev polynomials and the Catalan numbers arising in studying commuting difference operators. These generalizations are mostly related with ideas proposed by N. H. Abel in his publication in 1826, which then were reproduced by many authors in a modern language. As generalization of Chebyshev polynomials, we propose to consider polynomials with exactly two critical values well-studied in a so-called theory of dessins d’enfants. The Catalan numbers are located in the first column of the table of Harer-Zagier numbers related with the distribution by genus of orientable sewing of polygons with even number of sides. The commuting difference operators are implicitly contained in the Abel theory, who studied quasi-elliptic integrals, namely, the elliptic integrals of 3rd kind integrable in terms of logarithms. In the present work we formulate conjectures on relation between the main Abel theorem and commuting semi-infinite matrices. In the work we provide calculations supporting the conjectured relations.

MSC:
39A70 Difference operators
33C75 Elliptic integrals as hypergeometric functions

Keywords:
Chebyshev polynomials; Catalan numbers; Harer-Zagier numbers; polynomial Pell equation; dessins d’enfants

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[1] N. H. Abel, “Sur l’intégration de la formule différentielle \( \int \frac{\rho \, \text{d}z}{\sqrt{R}}, R \) et \( \rho \) etant des fonctions entières”, J. Reine Angew. Math., 1 (1826), 185-221

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