Summary: The paper is devoted to studying systems of semi-discrete equations \( \vec{r}_{n+1,x} = \vec{h}(x, n, \vec{r}_n, \vec{r}_{n+1}, \vec{r}_{n,x}) \) within the framework of an approach based on the concept of a characteristic Lie ring. Here \( \vec{r}_n = (r_{1n}, r_{2n}, \ldots, r_{Nn}) \), \( \vec{h} = (h^1, h^2, \ldots, h^N) \), \( n \in \mathbb{Z} \). Among integrable nonlinear partial differential equations and systems, we find Darboux integrable nonlinear hyperbolic equations and systems. A feature of such equations is the existence of integrals along each characteristic direction, the so-called \( x- \) and \( y-\)integrals.

This allows us to reduce the integration of a partial differential equation to integrating a system of ordinary differential equations. Darboux integrable equations and systems can be efficiently studied and classified by means of characteristic Lie rings. Papers by Leznov, Smirnov, Shabat, Yamilov underlie an algebraic approach for studying nonlinear hyperbolic systems. Currently, the algebraic approach is extended to semi-discrete and discrete equations. In this paper, we prove that the system has \( N \) essentially independent \( x-\)integrals if and only if the characteristic Lie ring corresponding to a continuous characteristic direction is finite-dimensional.

MSC:

37K10 Completely integrable infinite-dimensional Hamiltonian and Lagrangian systems, integration methods, integrability tests, integrable hierarchies (KdV, KP, Toda, etc.)
37K30 Relations of infinite-dimensional Hamiltonian and Lagrangian dynamical systems with infinite-dimensional Lie algebras and other algebraic structures
37D99 Dynamical systems with hyperbolic behavior

Keywords:

semi-discrete system of equations; characteristic ring; \( x \)-integral; Darboux integrable system

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