Pitowsky, Itamar

Correlation polytopes: Their geometry and complexity. (English) Zbl 0741.90054

The author is dealing with the facial structure of the correlation polytope $P$, which can be defined as follows: for an integer $n$ let $S \subseteq K_n = \{ (i, j) \mid 1 \leq i < j \leq n \}$ and

$$R(n, S) := \{ u(\varepsilon) = (u_1(\varepsilon), \ldots, u_n(\varepsilon), u_{12}(\varepsilon), \ldots, u_{ij}(\varepsilon), \ldots, u_{n-1n}(\varepsilon) \mid u_i(\varepsilon) = \varepsilon \text{ for } i = 1, \ldots, n; u_{ij}(\varepsilon) = \varepsilon_i \cdot \varepsilon_j \text{ for all } (i, j) \in S, \varepsilon = (\varepsilon_1, \ldots, \varepsilon_n) \in \{0, 1\}^n \}$$

a set of vectors corresponding to $S$; then $P(n, S)$ is the convex hull of $R(n, S)$ and especially $P$ the convex hull of $R(n, K_n)$.

It is shown that $P(n, S)$ has a non-empty interior and that its facial structure has a large symmetric group; thus one gets an exponential number of different facets by application of the symmetries.

Some facets are determined. Furthermore the author proves that the corresponding decision problem (given a vector $p \in \mathbb{R}_+^{n(n+1)/2}$; is $p \in P$?) is $NP$-complete. This means that unless $NP = co-NP$ deriving all the inequalities for $P$ is an “impossible” task.

The name “correlation polytope” stems from the fact that every point of $P(n, S)$ can be interpreted in terms of certain probability assignments.

Reviewer: H. Noltemeier (Würzburg)

MSC:

- 90C27 Combinatorial optimization
- 52B12 Special polytopes (linear programming, centrally symmetric, etc.)
- 90C60 Abstract computational complexity for mathematical programming problems

Keywords:

- facial structure
- correlation polytope

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References:


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