Summary: Let $R$ be a ring and $M$ be an $R$-module. In this paper we investigate modules $M$ such that every (simple) cosingular $R$-module is $M$-projective. We prove that every simple cosingular module is $M$-projective if and only if for $N \leq T \leq M$, whenever $T/N$ is simple cosingular, then $N$ is a direct summand of $T$. We show that every simple cosingular right $R$-module is projective if and only if $R$ is a right GV-ring. It is also shown that for a right perfect ring $R$, every cosingular right $R$-module is projective if and only if $R$ is a right GV-ring. In addition, we prove that if every $\delta$-cosingular right $R$-module is semisimple, then $\overline{Z}(M)$ is a direct summand of $M$ for every right $R$-module $M$ if and only if $Z_4(M)$ is a direct summand of $M$ for every right $R$-module $M$.

MSC:

16D10 General module theory in associative algebras
16D40 Free, projective, and flat modules and ideals in associative algebras
16D80 Other classes of modules and ideals in associative algebras

Keywords:

projective module; cosingular module; $\delta$-cosingular module; GV-ring

References:

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