Some special differential subordinations.

Summary: For an analytic function \( p \) satisfying \( p(0) = 1 \), we obtain sharp estimates on \( \beta \) such that the first order differential subordination

\[ p(z) + \beta z p'(z) \prec P(z) \text{ or } 1 + \beta z p'(z)/p^j(z) \prec P(z), \quad j = 0, 1, 2 \]

implies

\[ p(z) \prec Q(z) \]

where \( P \) and \( Q \) are Carathéodory functions. The key tools in the proof of main results are the theory of differential subordination and some properties of hypergeometric functions. Further, these subordination results immediately give sufficient conditions for an analytic function \( f \) to be in various well-known subclasses of starlike functions.

MSC:

30C45 Special classes of univalent and multivalent functions of one complex variable (starlike, convex, bounded rotation, etc.)
30C80 Maximum principle, Schwarz’s lemma, Lindelöf principle, analogues and generalizations; subordination

Keywords:

differential subordination; starlike function; lemniscate of Bernoulli; sine; Janowski function; exponential function; rational function; hypergeometric function

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References:


