Summary: We develop a randomized approximation algorithm for the size of set union problem $|A_1 \cup A_2 \cup \ldots \cup A_m|$, which is given a list of sets $A_1, \ldots, A_m$ with approximate set size $m_i$ for $A_i$ with $m_i \in ((1-\beta_L)|A_i|, (1+\beta_R)|A_i|)$, and biased random generators with probability $\text{Prob}(x = \text{RandomElement}(A_i)) \in [1-\alpha_{L}, 1+\alpha_{R}]$ for each input set $A_i$ and element $x \in A_i$, where $i = 1, 2, \ldots, m$ and $\alpha_{L}, \alpha_{R}, \beta_{L}, \beta_{R} \in (0, 1)$.

The approximation ratio for $|A_1 \cup A_2 \cup \ldots \cup A_m|$ is in the range $[(1-\epsilon)(1-\alpha_{L}), (1+\epsilon)(1+\alpha_{R})]$ for any $\epsilon \in (0, 1)$. The complexity of the algorithm is measured by both time complexity and round complexity. One round of the algorithm has non-adaptive accesses to those RandomElement($A_i$) functions $1 \leq i \leq m$, and membership queries ($x \in A_i$?) to input sets $A_i$ with $1 \leq i \leq m$. Our algorithm gives an approximation scheme with $O(m \cdot (\log m)^2)$ running time and $O(\log m)$ rounds in contrast to the existing algorithm [1] that needs $\Omega(m)$ rounds in the worst case with $O((1+\epsilon)m/\epsilon^2)$ running time, where $m$ is the number of sets. Our algorithm gives a flexible tradeoff with time complexity $O(m^{1+\epsilon})$ and round complexity $O(\frac{1}{\epsilon})$ for any $\epsilon \in (0, 1)$. Our algorithm runs sublinear in time under certain condition that each element in $A_1 \cup A_2 \cup \ldots \cup A_m$ belongs to $m^a$ sets for any fixed $a > 0$, to our best knowledge, we have not seen any sublinear results about this problem. Our algorithm can handle input sets that can generate random elements with bias, and its approximation ratio depends on the bias. We prove that it is $\#P$-hard to count the number of lattice points in a set of balls, and we also show that there is no polynomial time approximation algorithm for the maximal coverage problem with balls.

MSC: 68Qxx Theory of computing

Keywords: $\#P$-hard; randomized approximation; lattice points; rounds; sublinear time

Full Text: DOI

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