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A note about the norm of the sum and the anticommutator of two orthogonal projections.  
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Summary: In this note, we prove that for any two orthogonal projections $P_T, P_S$ on a Hilbert the well-known norm formulas

$$
\|P_T + P_S\| = 1 + ||P_T P_S||,
$$

unless $P_T = P_S = 0$ and

$$
\|P_T P_S + P_S P_T\| = ||P_T P_S||^2 + ||P_T P_S||,
$$
can be derived from each other. Such result is obtained from the relation between the spectra of the sum and product of any two idempotents in a Banach algebra. Applications of our results are given.

MSC:

47Axx General theory of linear operators  
47Bxx Special classes of linear operators  
46Lxx Selfadjoint operator algebras ($C^*$-algebras, von Neumann ($W^*$-) algebras, etc.)

Keywords:

orthogonal projection; norm inequalities; commutator; anticommutator