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Finite étale extensions of Tate rings and decompletion of perfectoid algebras. (English)
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Summary: In this paper, we examine the behavior of ideal-adic separatedness and completeness under certain ring extensions using trace map. Then we prove that adic completeness of a base ring is hereditary to its ring extension under reasonable conditions. We aim to give many results on ascent and descent of certain ring theoretic properties under completion. As an application, we give conceptual details to the proof of the almost purity theorem for Witt-perfect rings by Davis and Kedlaya. Witt-perfect rings have the advantage that one does not need to assume that the rings are complete and separated.

MSC:
13A18 Valuations and their generalizations for commutative rings
13B22 Integral closure of commutative rings and ideals
13B40 Étale and flat extensions; Henselization; Artin approximation
13F35 Witt vectors and related rings
13J10 Complete rings, completion
11S15 Ramification and extension theory
14G45 Perfectoid spaces and mixed characteristic

Keywords:
almost purity; completion; étale extension; non-Archimedean Banach ring; perfectoid algebra; Witt-perfect algebra